

Maths Knowledge Organiser @ Lenham Primary

Made with charisma

MRS CULVER MAR 19, 2021 11:55AM

Language

Addend

A number to be added to another

Aggregation

combining two or more quantities or measures to find a total

Array

An ordered collection of counters cubes or other items in rows and columns

Augmentation

increasing a quantity or measure by another quantity

Commutative

numbers can be added or multiplied in any order

Complement

in addition a number and its complement make a total e.g. 300 is the complement to 700 to make 1000

Difference

The numerical difference between two numbers is found by comparing the quantity in each group

Dividend

In division, the number that is divided.

Divisor

In division, the number by which another is divided

Exchange

Change a number or expression for another of an equal value

Factor

A number that multiplies with another to make a product

Minuend

A quantity or number from which another is subtracted

Multiplicand

In multiplication, a number to be multiplied by another

Partitioning

Splitting a number into its complement parts

Product

The result of multiplying one number by another

Quotient

The result of division

Reduction

Subtraction as take away

Remainder

The amount left over after a division when the divisor is not a factor of the dividend

Scaling

Enlarging or reducing a number by a given amount, called the scale factor

Subitise

Instantly recognise the number of objects in a small group without needing to count

Subtrahend

A number to be subtracted from another

Sum

The result of an addition

Total

The aggregate or the sum found by addition

Representations and Manipulatives

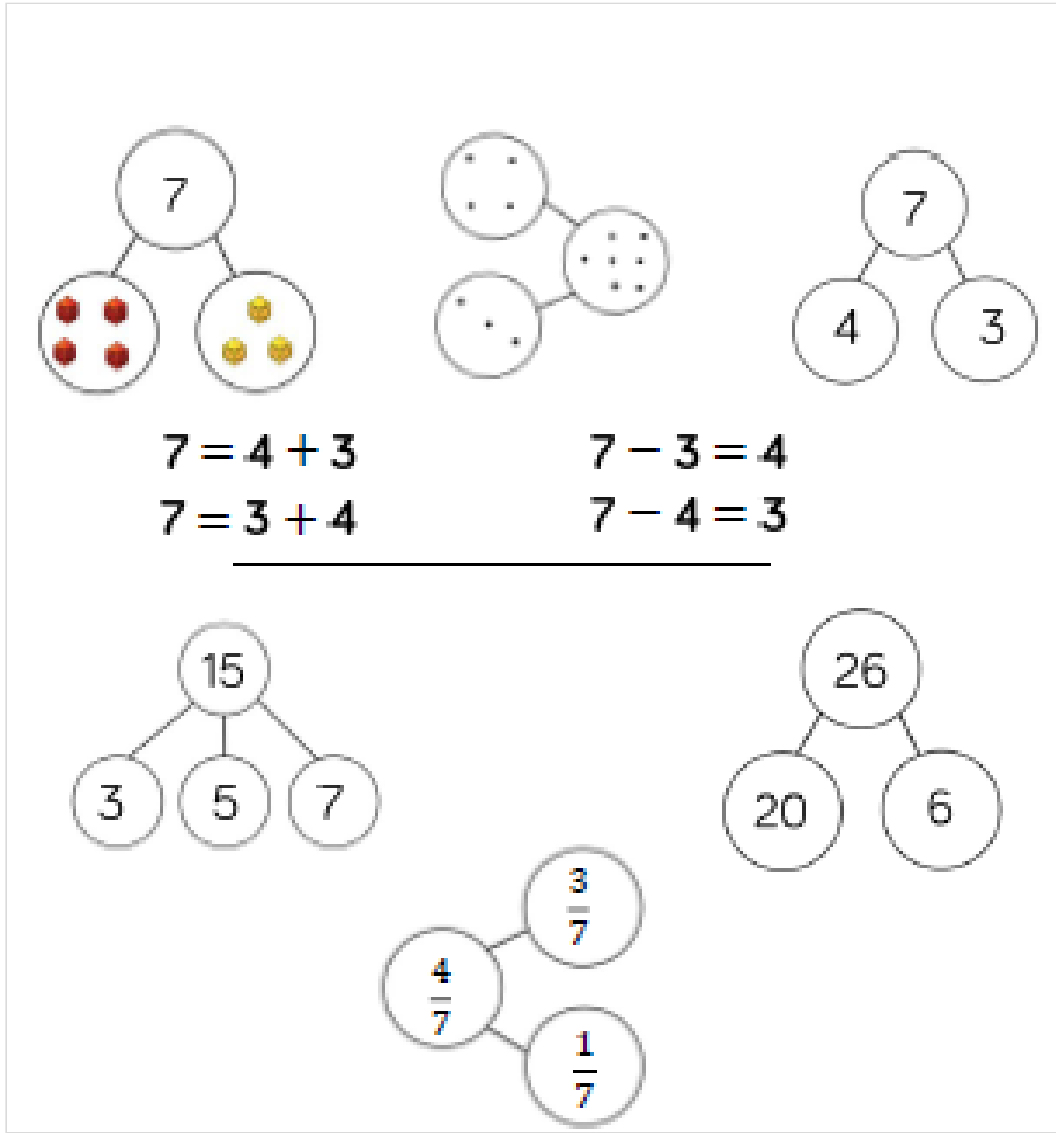
Part Part Whole

This **part-whole model** supports children in thier understanding of aggregation and partitioning.

When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts are empty, children use partitioning to find the missing part.

In KS2, children can apply their understanding of **part-whole model** to add and subtract fractions, decimals and percentages.



Bar model (single)

The single **bar model** is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line across as a concrete representation of the **bar model**.

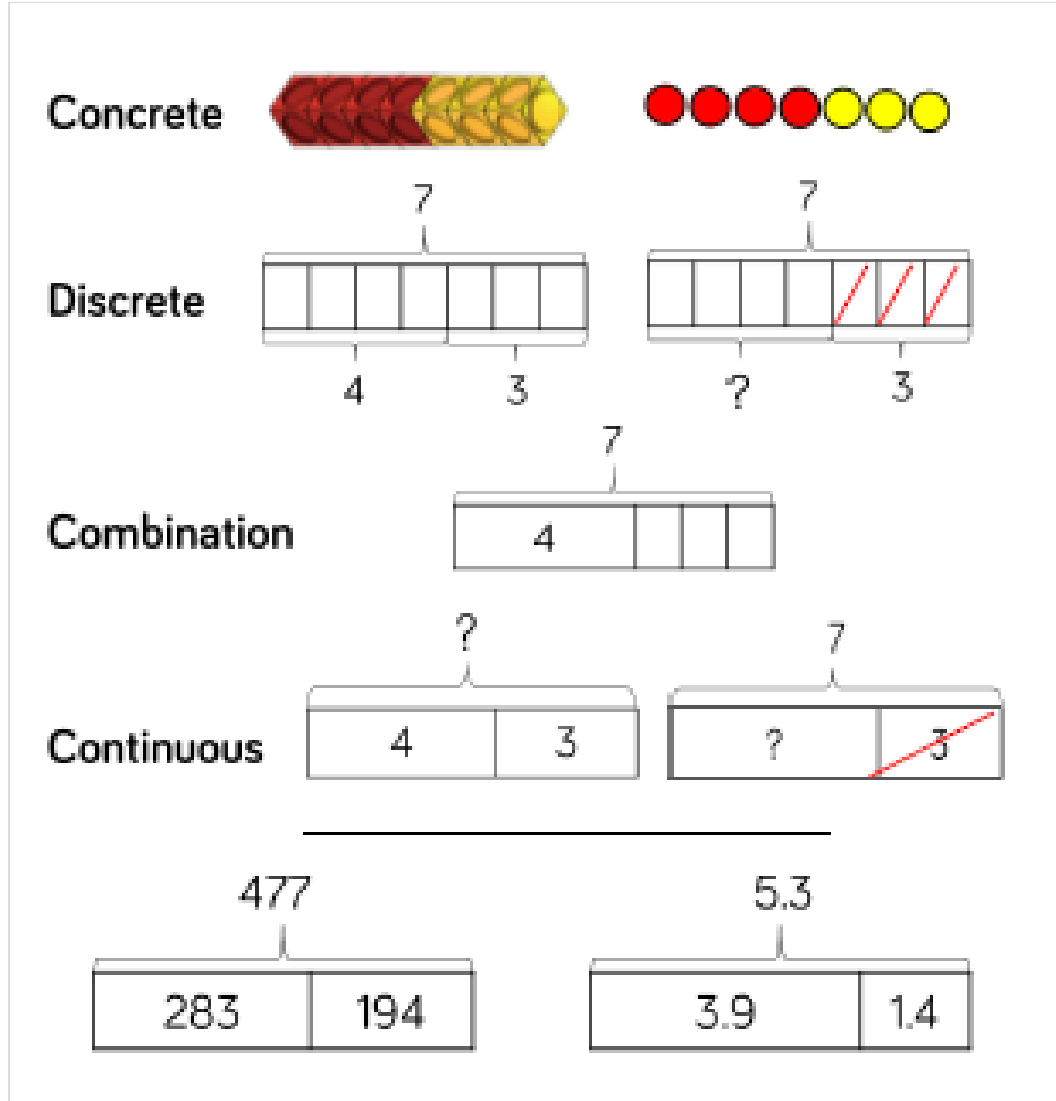
Discrete **bar models** are a good starting point with smaller numbers. Each box represents one

whole.

The combination **bar model** can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous **bar model**.

Continuous **bar models** are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

In KS2, children can use **bar models** to represent larger numbers, decimals and fractions.



Bar Model (multiple)

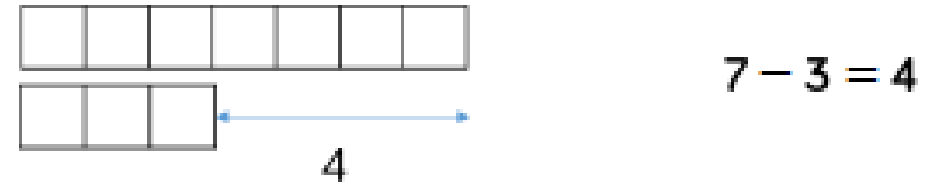
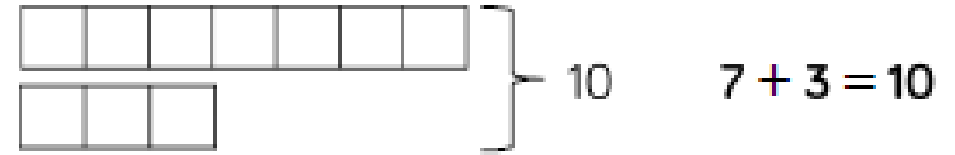
The **multiple bar model** is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete **bar model** whilst continuous bar models are more effective for larger numbers.

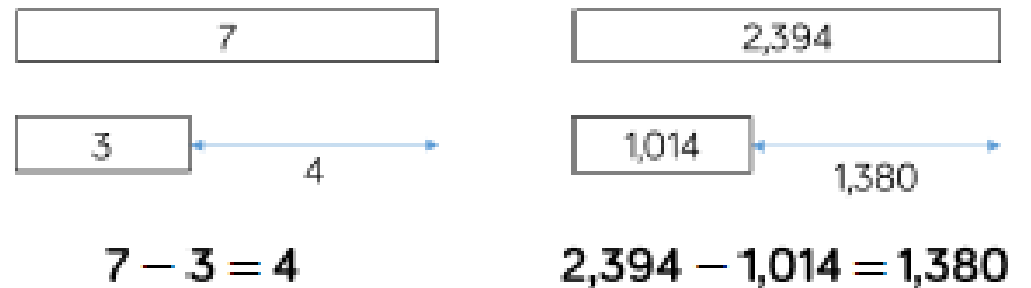
Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

Discrete



Continuous



Bar Model (Multiplication and Division)

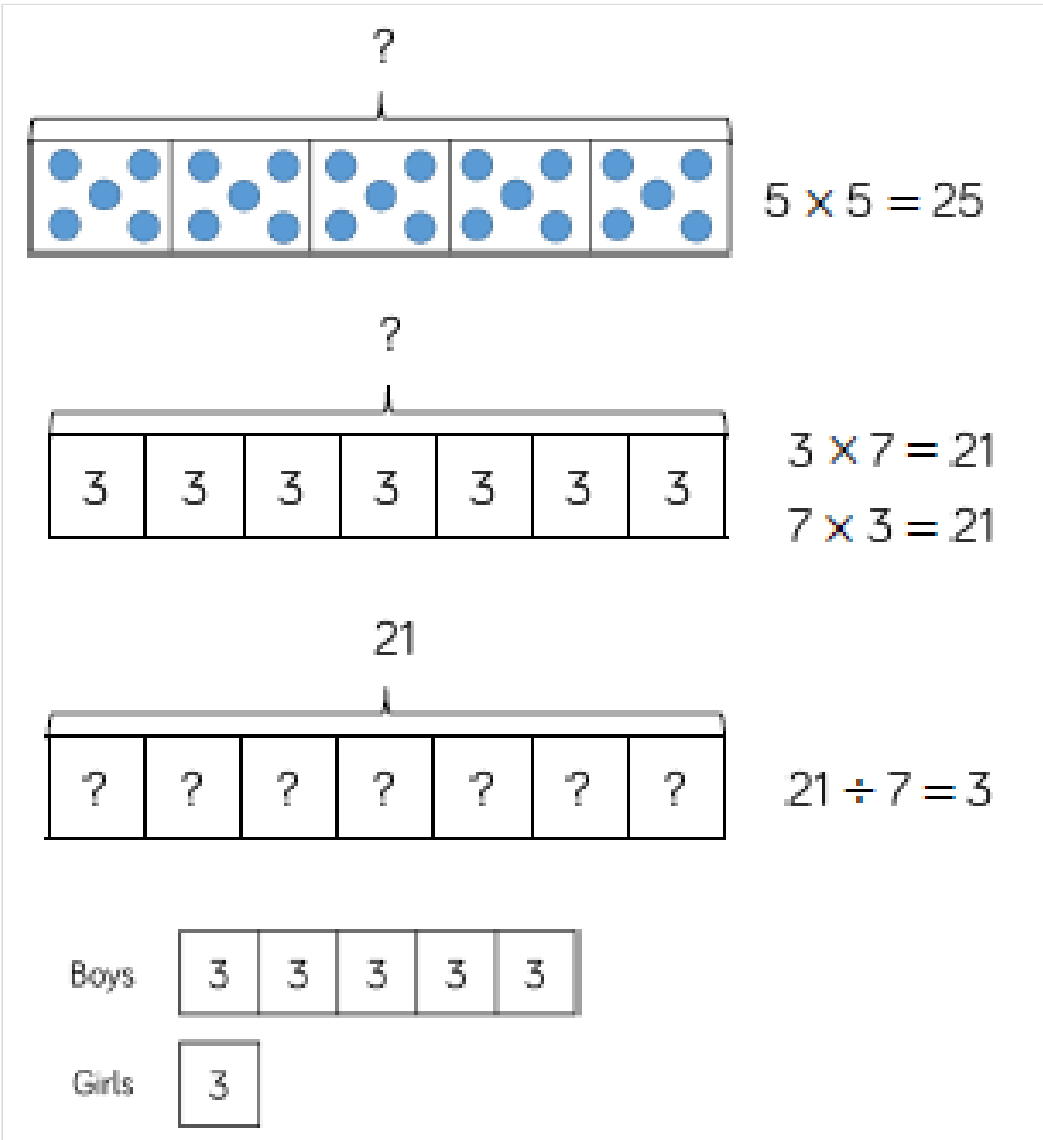
Children can use the **single bar model** to represent multiplication as repeated addition. They could use counters, cubes or dots within the **bar model** to support calculation before moving on to placing digits into the **bar model** to represent the multiplication.

Division can be represented by showing the total of the **bar model** and then dividing the **bar model** into equal groups.

It is important when solving word problems that the **bar model** represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one **bar model** is useful to represent this type of problem e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?

The multiple **bar model** provides an opportunity to compare the groups.



Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

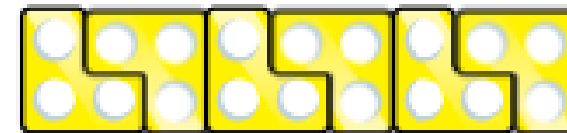
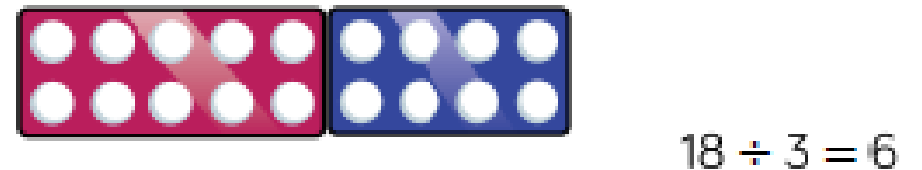
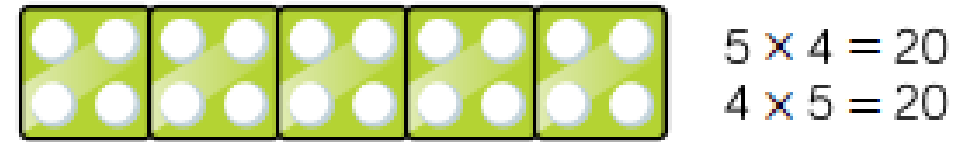
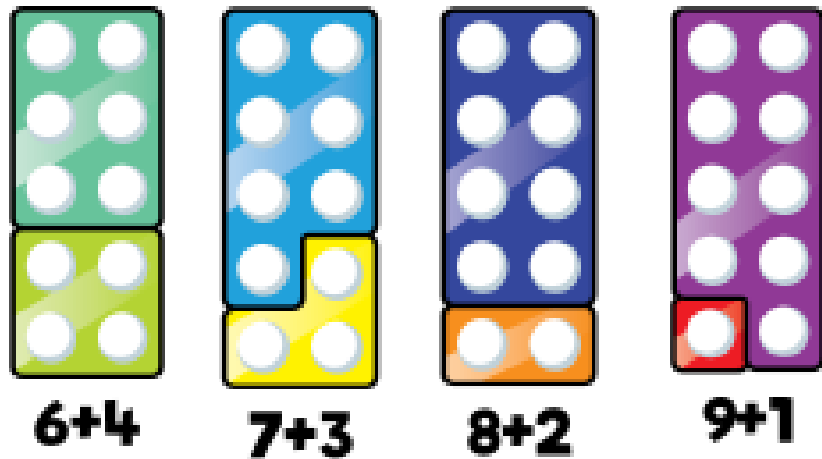
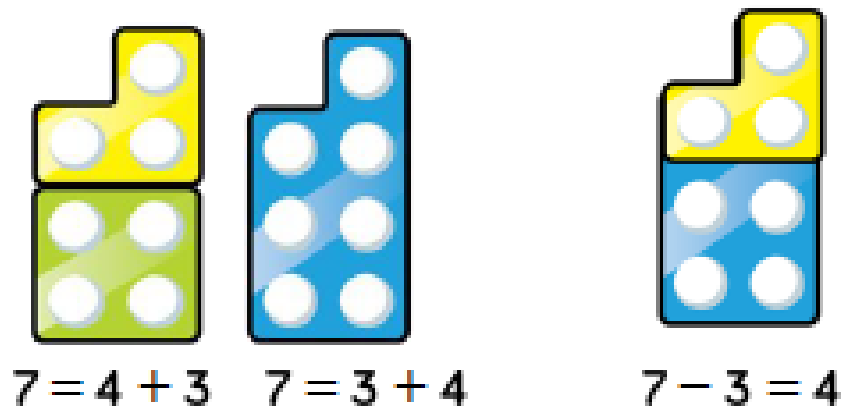
When adding numbers, children can see how the parts come together making a whole. As children use **number shapes** more often, they can also start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.

Number Shapes

When dividing, **number shapes** support children's understanding of division as grouping. Children make the number they are dividing and then place the **number shape** they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18.



Number Shapes (Multiplication and Division)

Number shapes support children's understanding of multiplication as repeated addition.

Children can build multiplications in a row using the **number shapes**. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens **number shapes** along with the other necessary shapes over the top of the row to check the total. Using the **number shapes** in multiplication can support children in discovering patterns of multiplication e.g. odd x odd = even, odd x even = odd, even x even = even.

Cubes

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

When adding numbers, children can see how the parts come together to make a whole. children could use two different colours of **cubes** to represent the numbers before putting them together to create the whole.

Ten Frames (within 10)

When subtracting numbers, children can start with the whole and then remove the number of **cubes** that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

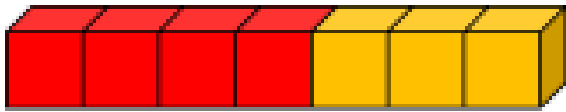
Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

When adding and subtracting within 10, the **ten frame** can support children to understand the different structures of addition and subtraction.

Using the language of parts and wholes represented by objects on the **ten frame** introduces children to aggregation and partitioning.

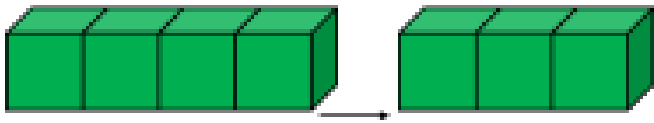
Children can also use **ten frames** to look at augmentation and take-away. This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change. e.g First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.



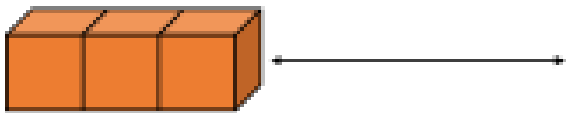
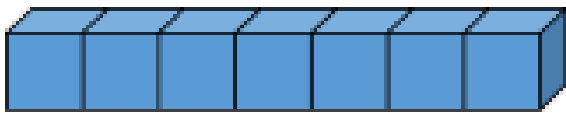
$$7 = 4 + 3$$



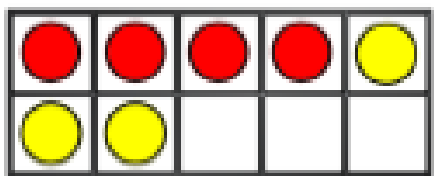
$$7 = 3 + 4$$



$$7 - 3 = 4$$



$$7 - 3 = 4$$



$$4 + 3 = 7$$

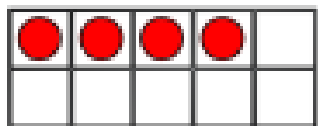
$$3 + 4 = 7$$

$$7 - 3 = 4$$

$$7 - 4 = 3$$

4 is a part.
3 is a part.
7 is the whole.

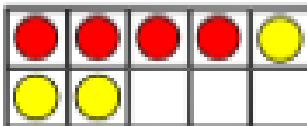
First



Then



Now



$$4 + 3 = 7$$

First



Then

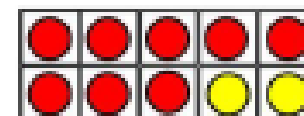


Now

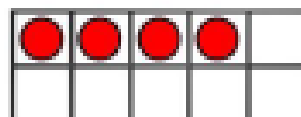
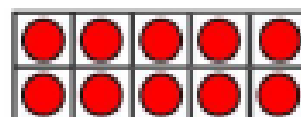


$$7 - 3 = 4$$

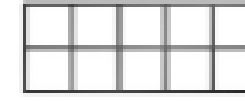
When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the **ten frames** support the link to effective mental methods of addition as well as the importance of commutativity.



$$8 + 7 = 15$$



$$14 - 6 = 8$$



$$7 + 6 + 3 = 16$$

Ten Frames (within 20)

When adding two single digits, children can make each number on separate **ten frames** before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 tens frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

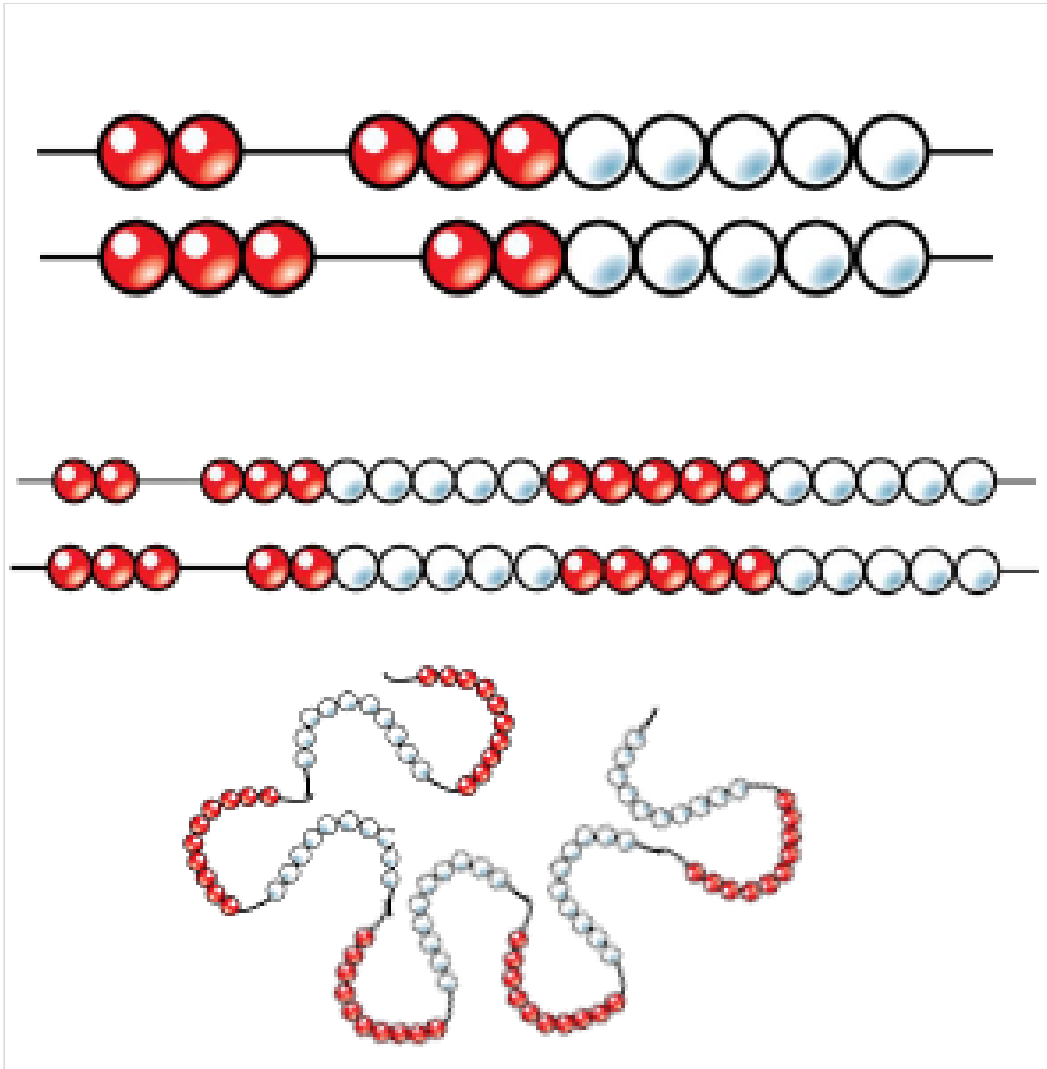
Bead Strings

Different sizes of **bead strings** can support children at different stages of addition and subtraction.

Bead strings to 10 are very effective at helping children to investigate number bonds up to 10. They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. $2 + 8 = 10$, move one bead $3 + 7 = 10$.

Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.

Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. **Bead strings** can show a link to adding to the next 10 on number lines which supports a mental method of addition.



Bead Strings (Multiplication and Division)

Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently. Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20

Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads in to the number they are dividing by e.g. 20 divided by 4 - Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.

The image shows three examples of bead strings used for multiplication and division. Each example consists of a horizontal string of beads and two equations below it.

- Example 1:** A string with 15 red beads followed by 5 white beads. Below it are the equations $5 \times 3 = 15$ and $3 \times 5 = 15$ on the left, and $15 \div 3 = 5$ on the right.
- Example 2:** A string with 15 red beads followed by 5 white beads. Below it are the equations $5 \times 3 = 15$ and $3 \times 5 = 15$ on the left, and $15 \div 5 = 3$ on the right.
- Example 3:** A string with 20 red beads followed by 10 white beads. Below it are the equations $4 \times 5 = 20$ and $5 \times 4 = 20$ on the left, and $20 \div 4 = 5$ on the right.

Number tracks

Number tracks are useful to support children in their understanding of addition.

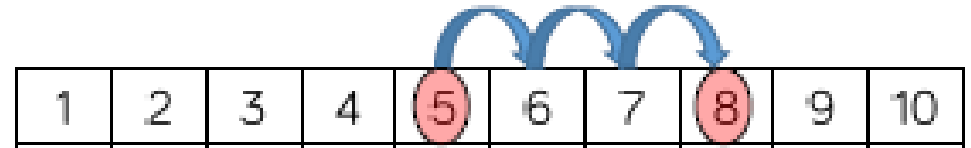
When adding, children count on to find the total of the numbers. On a **number track**, children can place a counter on the starting number and then count on to find the total.

When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

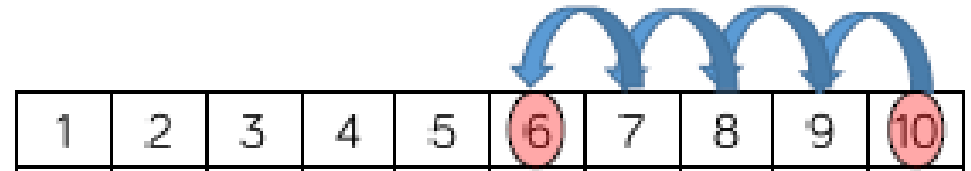
Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

Playing board games can help children to become familiar with the idea of counting on using a **number track** before they move on to number lines.

$$5 + 3 = 8$$



$$10 - 4 = 6$$



$$8 + 7 = 15$$



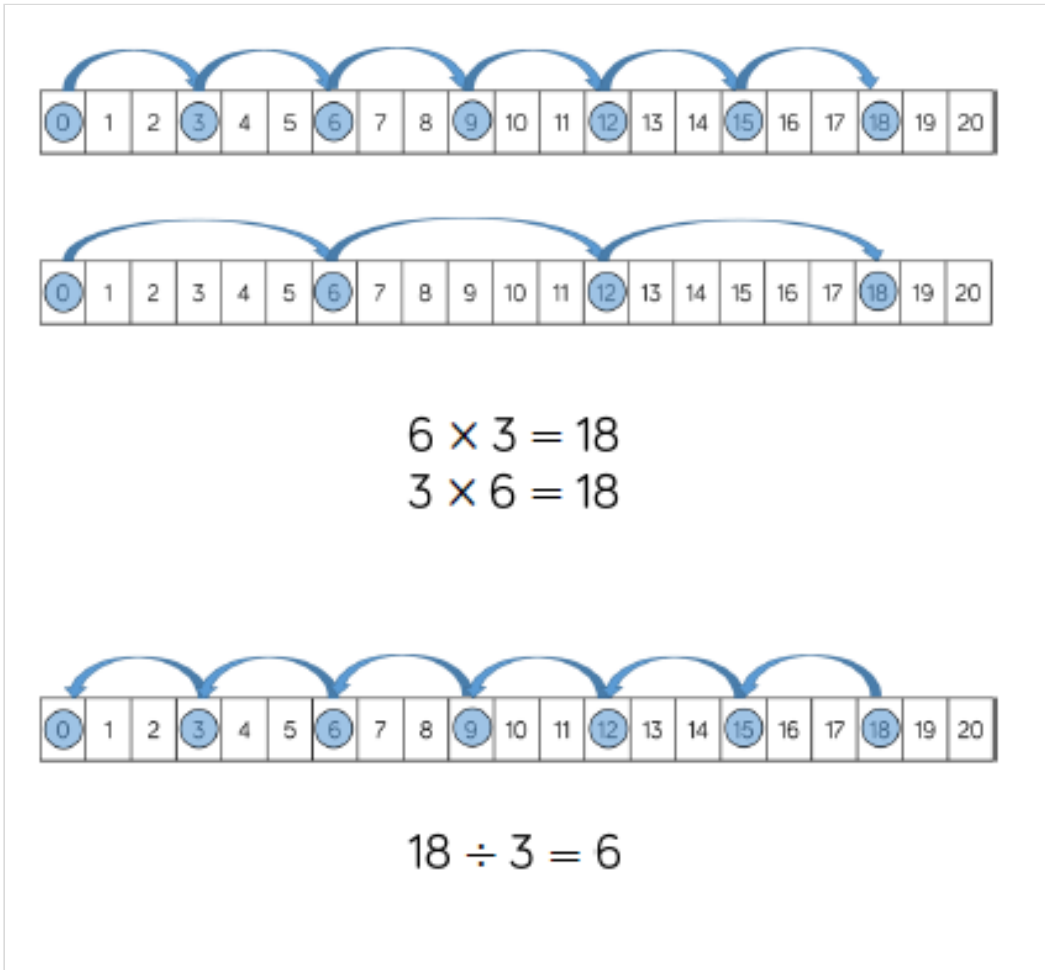
Number Tracks (Multiplication and Division)

Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the **number track** can support children to keep track of their counting.

When multiplying children place their counter on 0 to start and then count on to find the product of the numbers.

When dividing, children place their counter on the number they are dividing and then count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.



Number Lines (labelled)

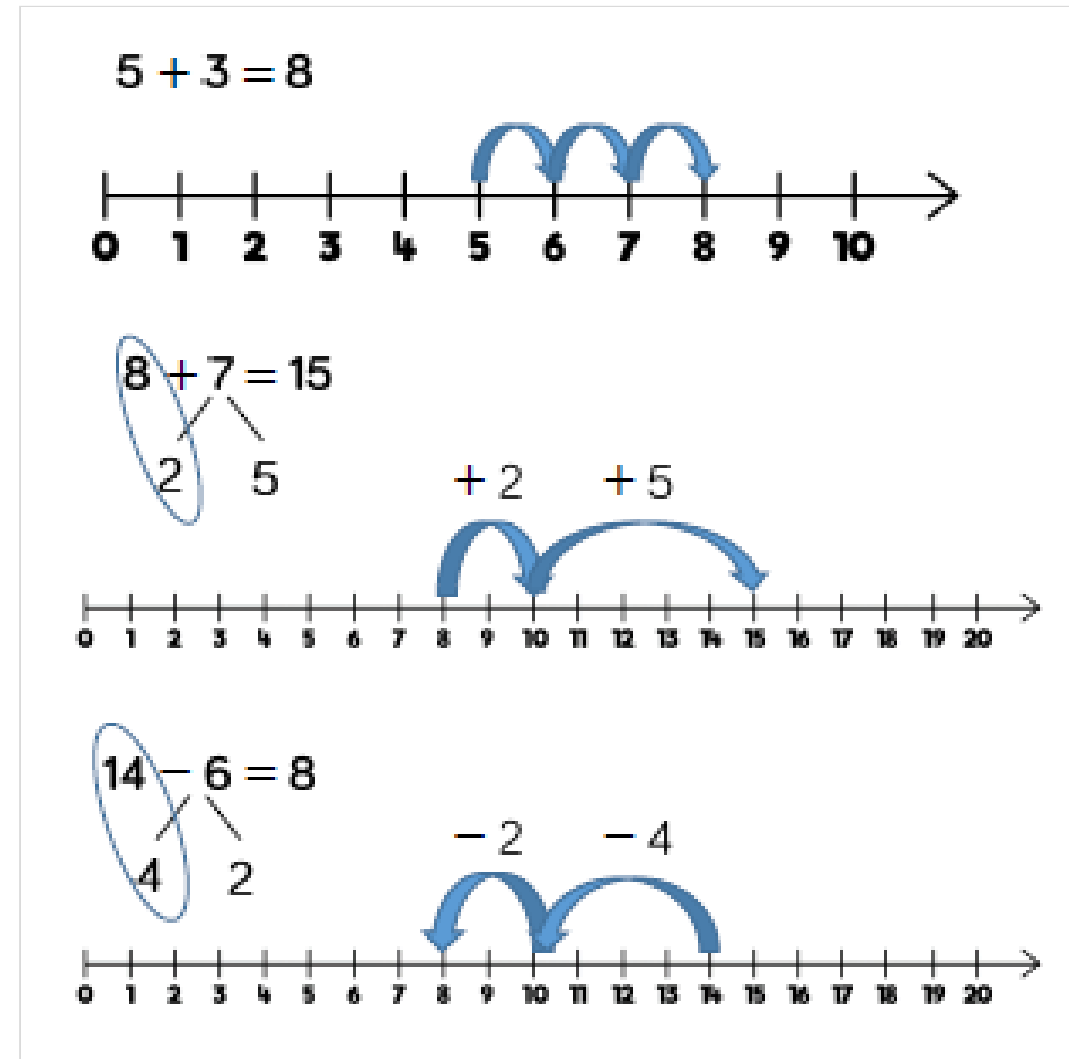
Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by **ten frames**.

The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by **ten frames** so children can see how they partition the smaller number into the two separate jumps.

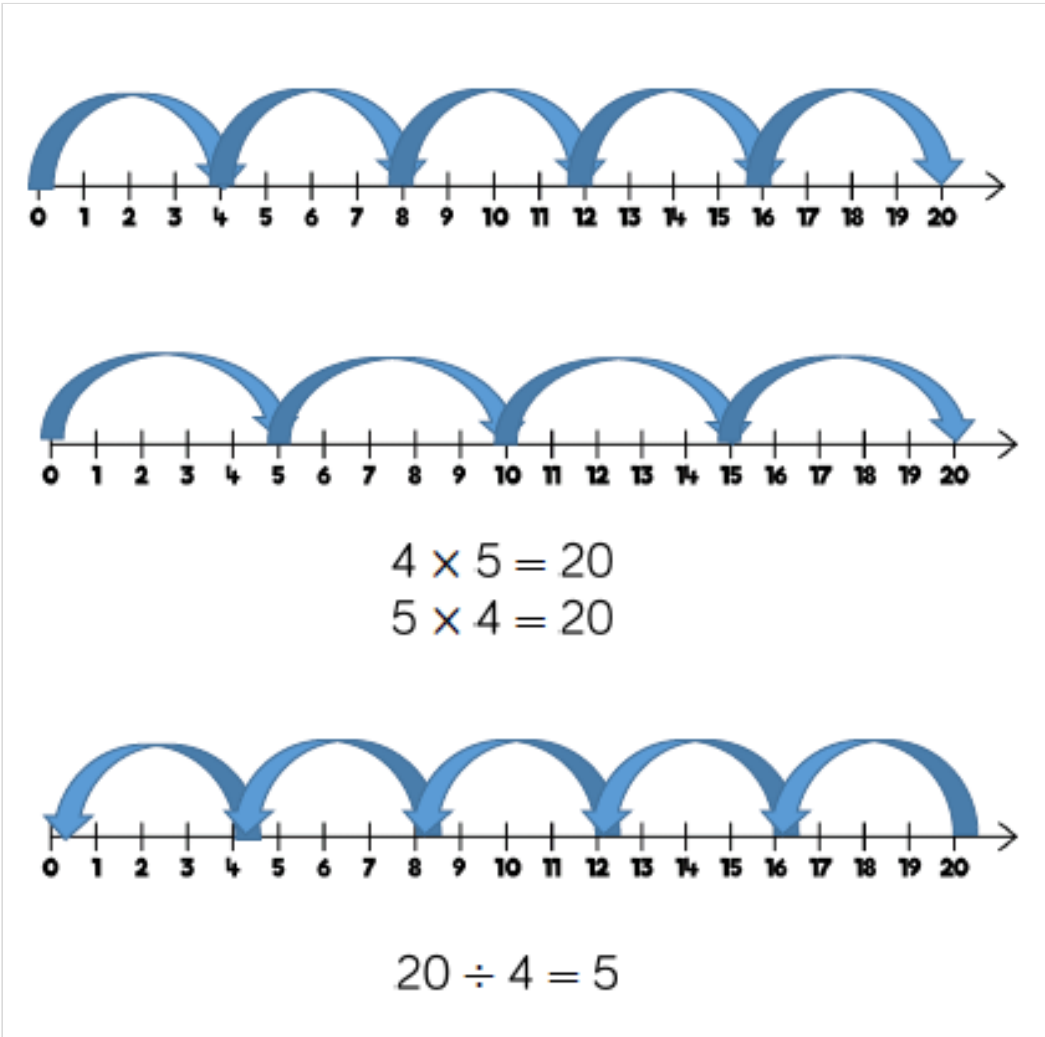


Number Lines (Multiplication and Division)

Labelled **number lines** are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

When multiplying, children start at 0 and then count on to find the product of the numbers. When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.



Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

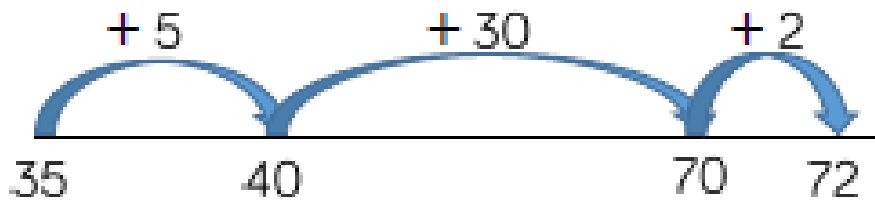
Developing from **labelled number lines**, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

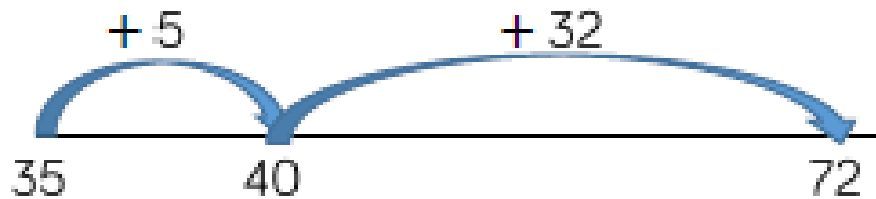
Blank number lines can also be used effectively to help children subtract by finding the difference between numbers, This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

Number Lines (blank)

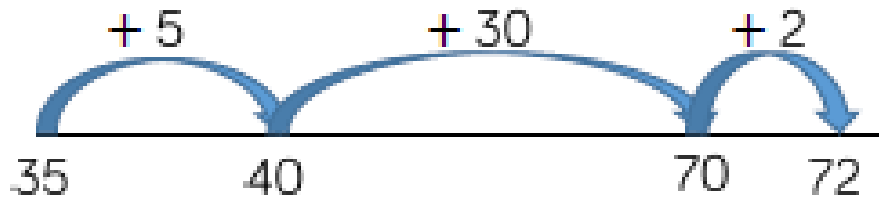
$$35 + 37 = 72$$



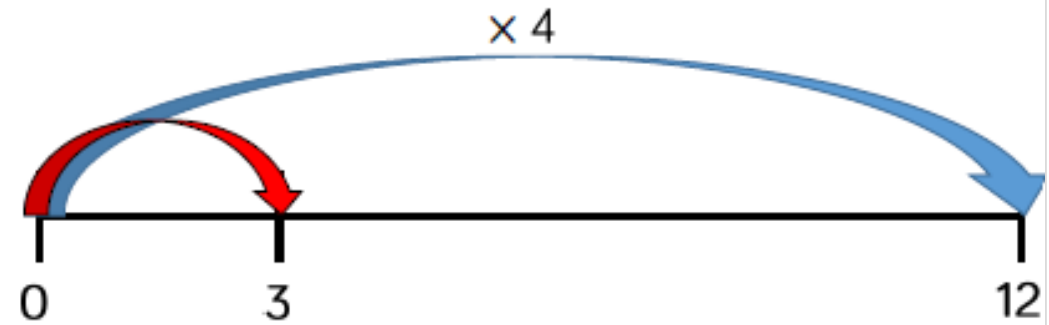
$$35 + 37 = 72$$



$$72 - 35 = 37$$



A red car travels 3 miles.
A blue car 4 times further.
How far does the blue car travel?



A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

Number Lines - Blank (Multiplication and Division)

Children can use **blank number lines** to represent scaling as multiplication or division.

Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

Blank number lines without intervals can also be used for children to represent scaling.

Straws

Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

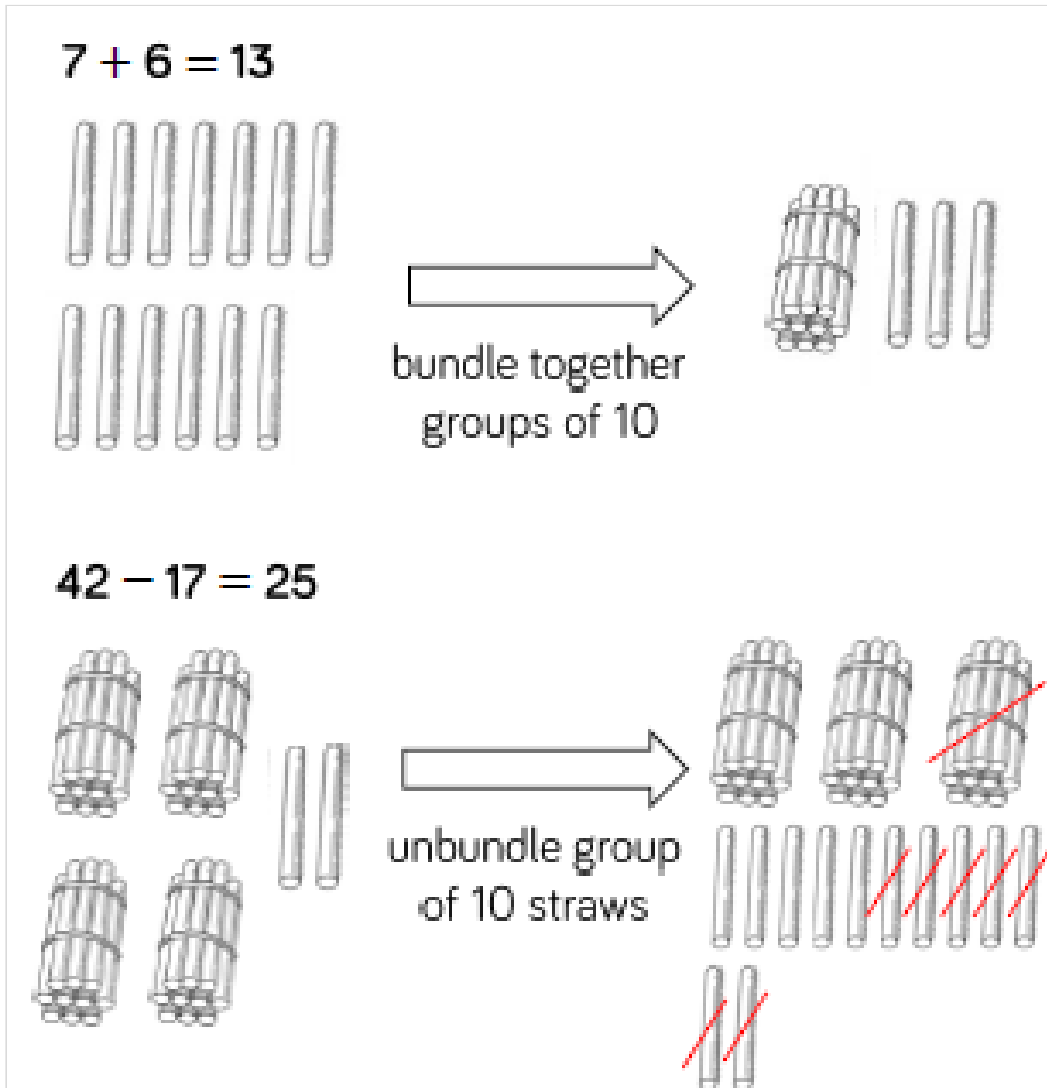
Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten **straws**.

When adding number, children bundle a group of 10 **straws** to represent the exchange from 10 ones to 1 ten. They then add the individual **straws** (ones) and bundles of **straws** (tens) to find

the total.

When subtracting numbers, children unbundle a group of 10 **straws** to represent the exchange from 1 ten to 10 ones.

Straws provide a good stepping stone to adding and subtracting using **Base 10/Dienes**



Using **Base 10 or Dienes** is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing **Base 10** so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange. The representation becomes less efficient with larger numbers due to the size of **Base 10**. In this case, **Place Value Counters** may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children.

How many ones are there altogether?

Can we make an exchange? (Yes or No)

How many do we exchange (10 ones for ten, show exchanged 10 in tens column by writing 1 in column)

How many ones do we have left? (Write in ones column)

Repeat for each column

Base 10/Dienes (Addition)

Tens	Ones
	●●●● ●●●●
	●●●●

$$\begin{array}{r} 38 \\ + 23 \\ \hline 61 \\ \hline 1 \end{array}$$

Hundreds	Tens	Ones
■ ■		●●●●
■		●●●●

$$\begin{array}{r} 265 \\ + 164 \\ \hline 429 \\ \hline 1 \end{array}$$

column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

Tens	Ones
	●●●●
	●●●●

$$\begin{array}{r} 5 \quad 1 \\ \cancel{6}5 \\ - 28 \\ \hline 37 \end{array}$$

Hundreds	Tens	Ones
■ ■		●●●●
■		●●●●

$$\begin{array}{r} 3 \quad 1 \\ \cancel{4}35 \\ - 273 \\ \hline 262 \end{array}$$

Base 10/Dienes (Subtraction)

Using **Base 10 or Dienes** is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing **Base 10** so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using **Base 10**, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the

Base 10/ Dienes (Multiplication)

Using **Base 10 or Dienes** is an effective way to support children's understanding of column multiplication. It is children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

As numbers become larger in multiplication or the amounts of groups becomes higher, **Base 10/Dienes** becomes less efficient due to the amount of equipment and number of exchanges needed.

Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces. This area model can be linked to the grid method or formal column method of multiplying 2-digits by 2-digits.

Hundreds	Tens	Ones
		●●●●
		●●●●
		●●●●

$$\begin{array}{r}
 24 \\
 \times 3 \\
 \hline
 72 \\
 \hline
 1
 \end{array}$$

Base 10/Dienes (Division)

Using **Base 10 or Dienes** is an effective way to move children from representing them as tens and ones in order to divide. Children can then share the **Base 10/Dienes** between different groups e.g. by drawing circles or by rows on a place value grid.

When they are sharing, children start with the larger place value and work left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the part-whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.

$68 \div 2 = 34$

Tens	Ones
	●●●●
	●●●●
	●●●●

$72 \div 3 = 24$







Place Value Counters (Addition)

Using **place value counters** is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing





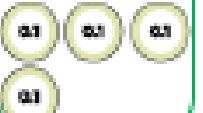

counters so they can see the clear links between the written method and the model.

Children should first add without exchange before moving on to addition with exchange. Different **place value counters** can be used to represent larger numbers or decimals. If you don't have **place value counters**, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

Hundreds	Tens	Ones
		
		

$$\begin{array}{r} 384 \\ + 237 \\ \hline 621 \\ 11 \end{array}$$







Ones	Tenths	Hundredths
		
		

$$\begin{array}{r} 3.65 \\ + 2.41 \\ \hline 6.06 \\ 1 \end{array}$$







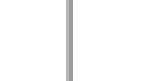

Using **place value counters** is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have **place value counters**, use normal counters on a place value grid to enable children to experience the exchange between columns.

When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g exchange 1 ten for 10 ones. They can then subtract efficiently.

Hundreds	Tens	Ones
		
		

$$\begin{array}{r} 52 \\ - 207 \\ \hline 445 \end{array}$$

Thousands	Hundreds	Tens	Ones
			
			

$$\begin{array}{r} 357 \\ - 2735 \\ \hline 1622 \end{array}$$

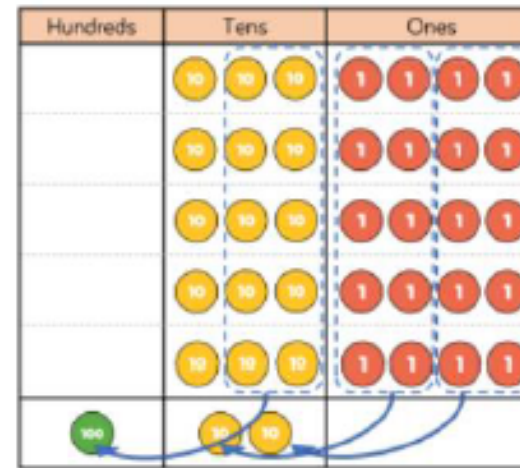
Place Value Counters (Subtraction)

Place Value Counters (Multiplication)

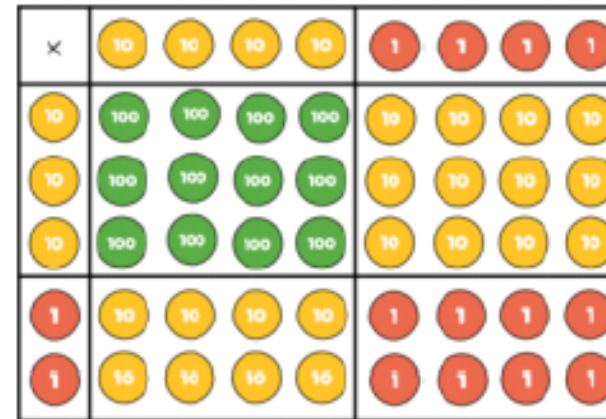
Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10/Dienes becomes less efficient due to the amount of equipment and number of exchanges needed. The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the are model of multiplication well. Children can see how to multiply 2-digit numbers by 2-digit numbers.



$$\begin{array}{r} 34 \\ \times 5 \\ \hline 170 \\ \hline 12 \end{array}$$



$$\begin{array}{r} 44 \\ \times 32 \\ \hline 8 \\ 80 \\ 120 \\ + 1200 \\ \hline 1408 \\ \hline 1 \end{array}$$

Addition

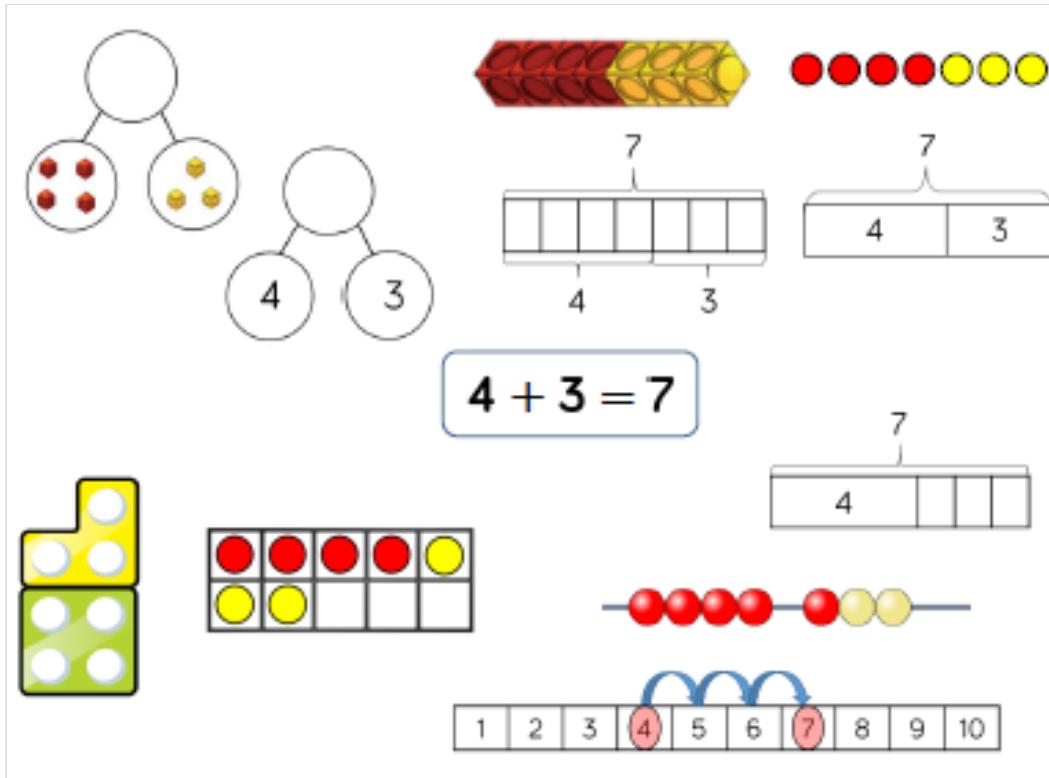
YEAR 1

Skill: Add 1-digit numbers within 10

When adding numbers to 10, children can explore both aggregation and augmentation.

The part-whole model, discrete and continuous bar model, number shapes and ten frame support aggregation.

The combination bar model, ten frame, bead string and number track all support augmentation.



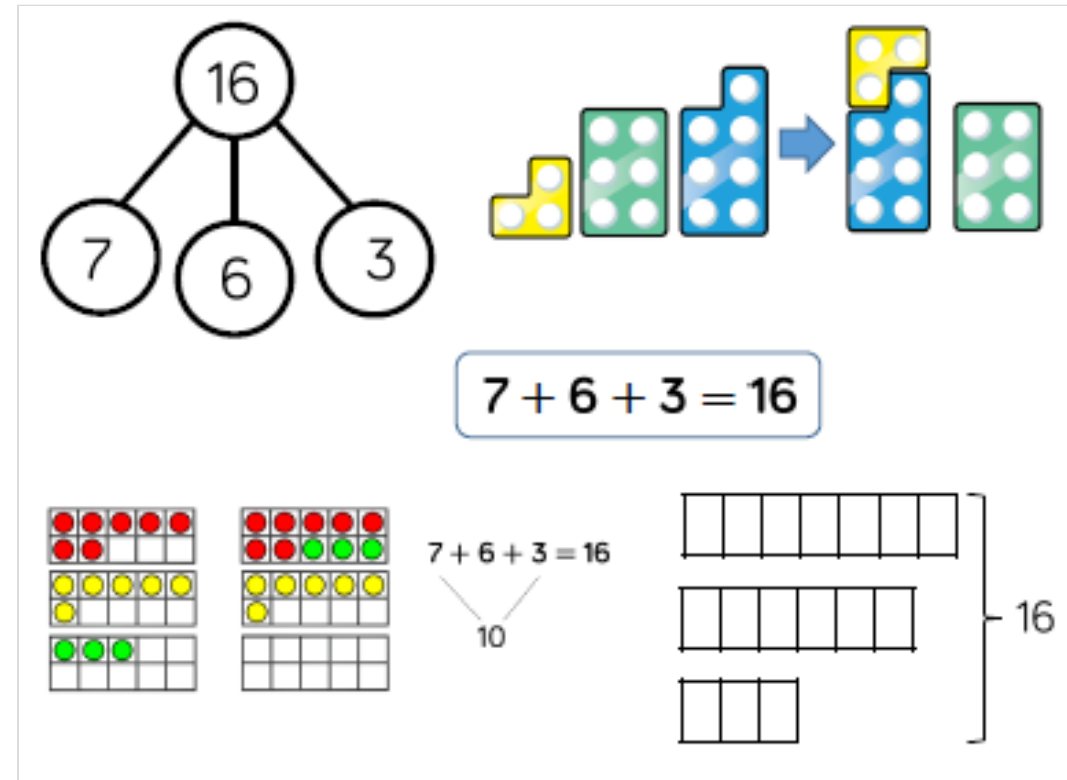
YEAR 2

Skill: Add three 1-digit numbers

When adding three 1-digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently.

This supports children in their understanding of commutativity.

Manipulatives that highlight number bonds to 10 are effective when adding three 1-digit numbers.

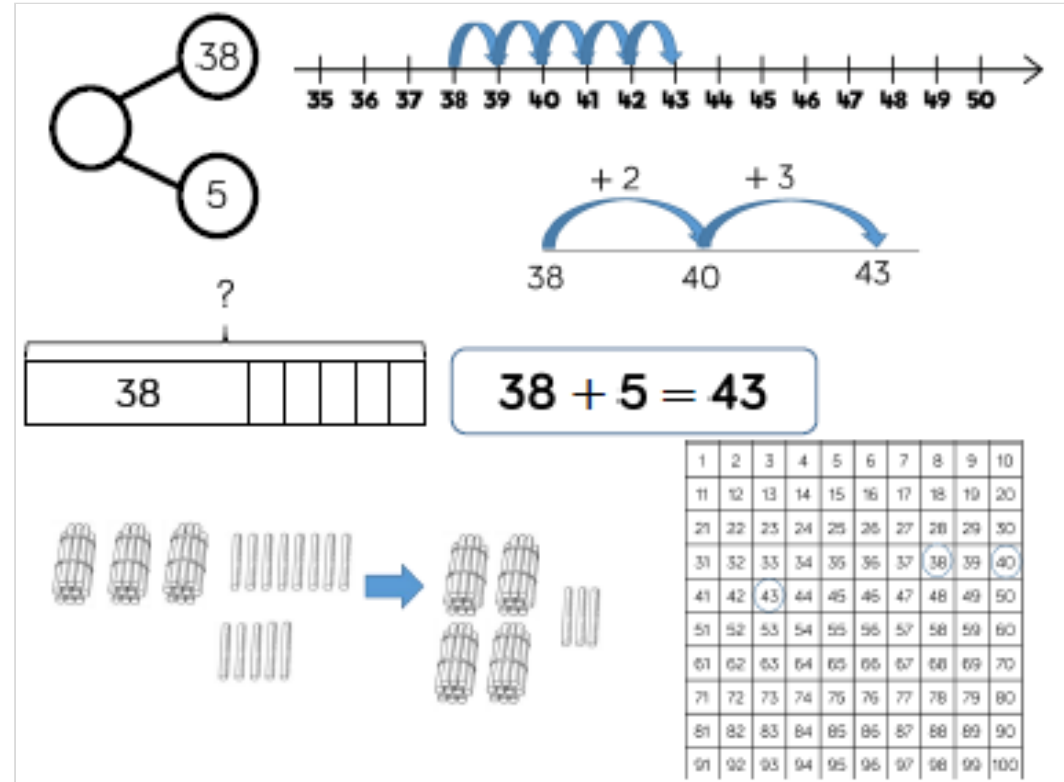
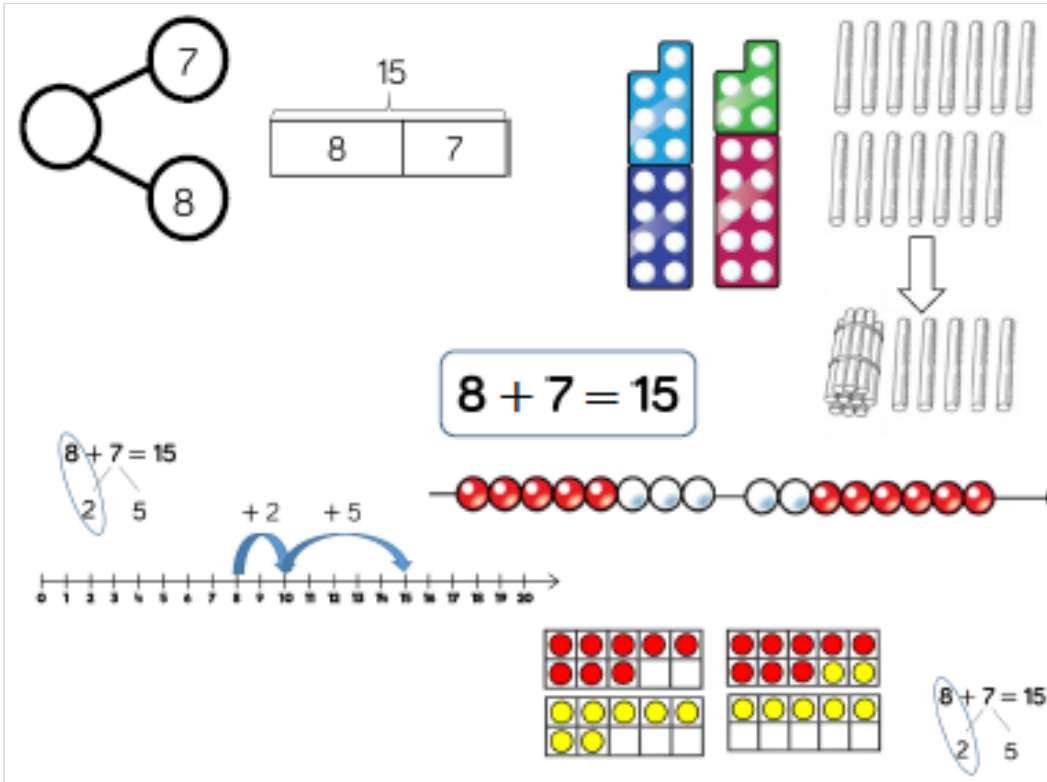


YEAR 1/2

Skill: Add 1 and 2-digit numbers to 20

When adding one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten.

Different manipulatives can be used to represent this exchange. Use concrete resources alongside number lines to support children in understanding how to partition their jumps.



YEAR 2/3

Skill: Add 1-digit and 2-digit numbers to 100

When adding single digits to a two-digit number, children should be encouraged to count on from the larger number.

They should also apply their knowledge of number bonds to add more efficiently e.g. $8 + 5 = 13$ so $38 + 5 = 43$

Hundred squares and straws can support children to find the number bond to 10.

YEAR 2/3

Skill: Add two 2-digit numbers to 100

At this stage, encourage children to use formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

Children can also use a blank number line to count on to find the total. Encourage them to jump to multiples of 10 to become efficient.

38

23

?

38 23

$38 + 23 = 61$

Tens	Ones
38	23
61	1

38
+ 23

61
1

Tens	Ones
38	23
61	1

YEAR 3

Skill: Add numbers with up to 3 digits

Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 3 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

265

164

?

265 164

265 164

$265 + 164 = 429$

Hundreds	Tens	Ones
265	164	
429	1	

265
+ 164

429
1

Hundreds	Tens	Ones
265	164	
429	1	

YEAR 4

Skill: Add numbers with up to 4 digits

Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 4 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

Diagram illustrating the addition of 1,378 and 2,148. The numbers are shown in a tree diagram, a box model, and a column method grid. The column method grid shows the numbers aligned by place value, with a carry of 1 from the tens column to the hundreds column. The final sum is 3,526.

Place value grids are shown below, using base ten blocks and counters to represent the numbers and the addition process. The first grid shows 1,378 using blue blocks. The second grid shows 2,148 using blue blocks. The third grid shows the sum 3,526 using blue blocks. The fourth grid shows the numbers 1,378 and 2,148 on a place value grid with counters, with arrows indicating the addition process and a carry of 1 from the tens column to the hundreds column.

1,378 + 2,148 = 3,526

YEAR 5/6

Skill: Add numbers with more than 4 digits

Place value counters or plain counters on a place value grid are the most effective concrete resources when adding numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using the column method to add larger numbers effectively.

Diagram illustrating the addition of 104,328 and 61,731. The numbers are shown in a tree diagram, a box model, and a column method grid. The column method grid shows the numbers aligned by place value, with a carry of 1 from the tens column to the hundreds column. The final sum is 166,059.

Place value grid is shown below, using counters to represent the numbers and the addition process. The grid shows 104,328 and 61,731 on a place value grid with counters, with arrows indicating the addition process and a carry of 1 from the tens column to the hundreds column.

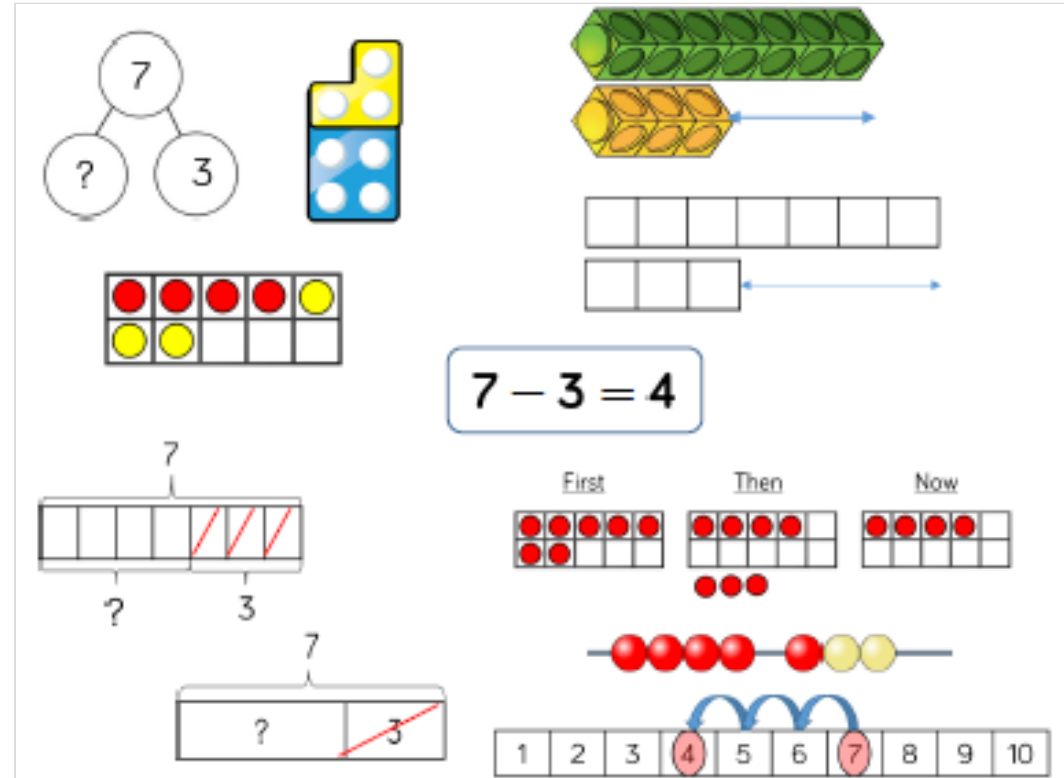
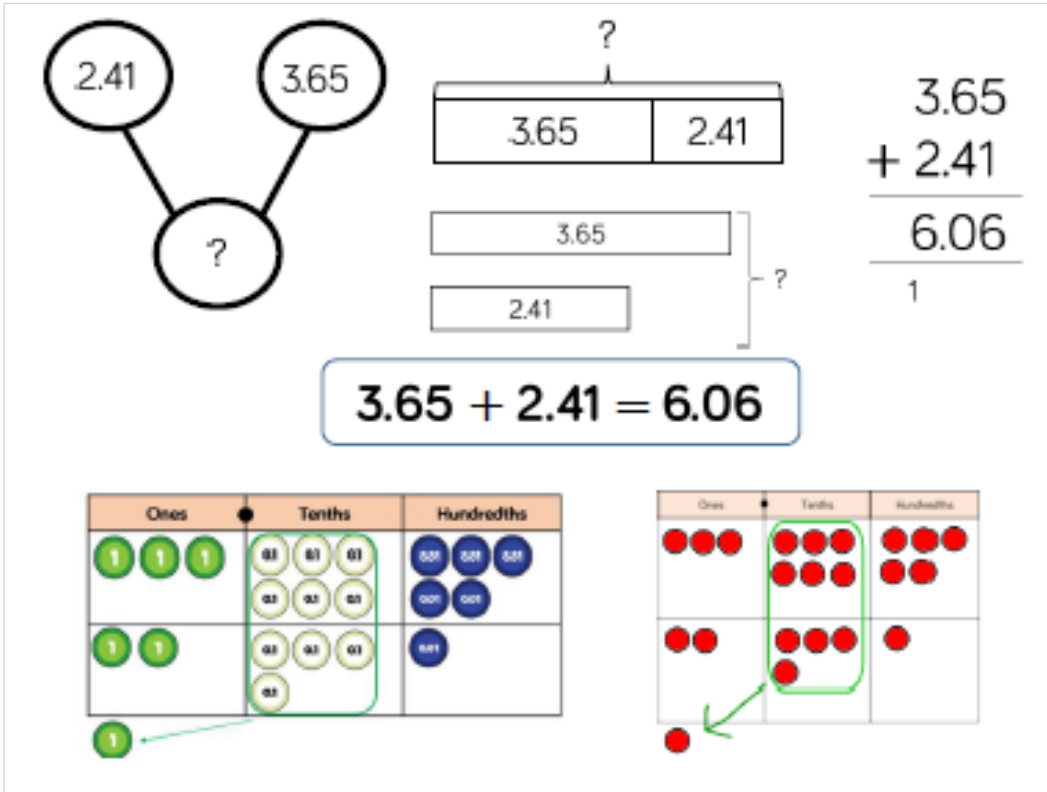
104,328 + 61,731 = 166,059

YEAR 5

Skill: Add with up to 3 decimal places

Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and 3 decimal places.

Ensure children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other measures.



Subtraction

YEAR 1

Skill: Subtract 1-digit numbers within 10

Part-whole models, bar models, ten frames and number shapes support partitioning.

Ten frames, number tracks, single bar models and bead strings support reduction.

Cubes and bar models with two bars can support finding the difference.

YEAR 1/2

Skill: Subtract 1 and 2-digit numbers to 20

When subtracting one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten.

Children should be encouraged to find the number bond to 10 when partitioning the subtracted number. Ten frames, number shapes and number lines are particularly useful for this.

6

14

14

6

8

14 - 6 = 8

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

14 - 6 = 8

4 2

-2 -4

14 - 6 = 8

4 2

65

28

65

28

65 - 28 = 37

28 30 60 65

+2 +30 +5

Tens Ones

65

-28

37

Tens Ones

10 10 10

1 1 1 1 1

YEAR 2

Skill: Subtract 1 and 2-digit numbers to 100

At this stage encourage children to use formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

Children can also use a blank number line to count on to find the difference. Encourage them to jump to multiples of 10 to become more efficient.

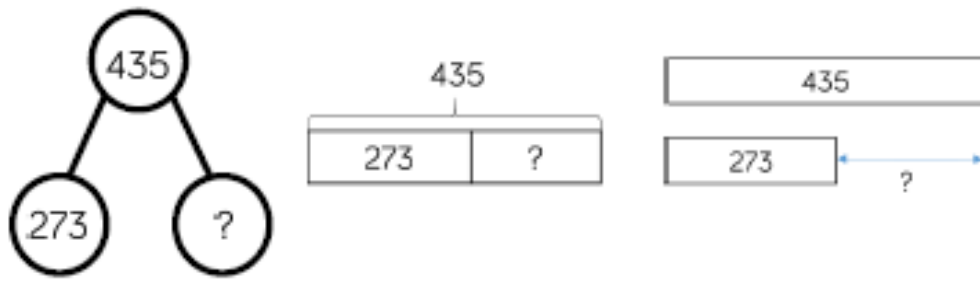
YEAR 3

Skill: Subtract numbers with up to 3 digits

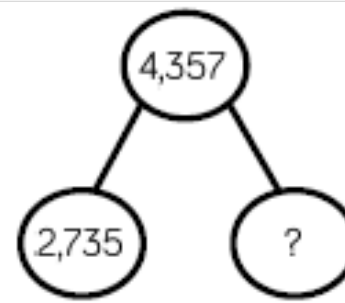
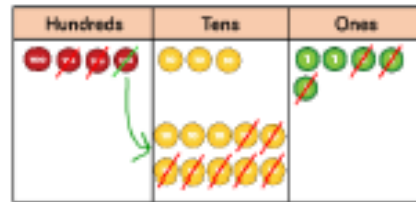
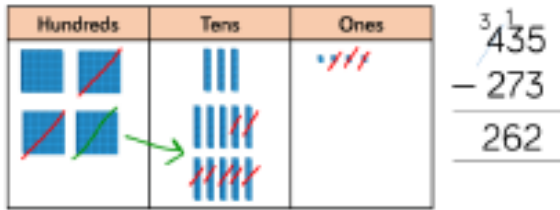
Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 3 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

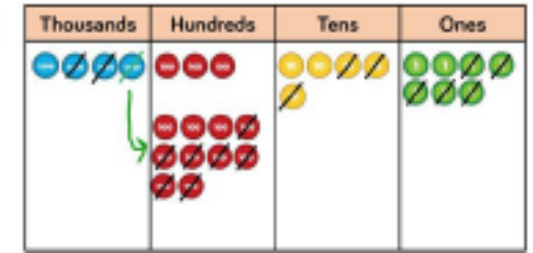
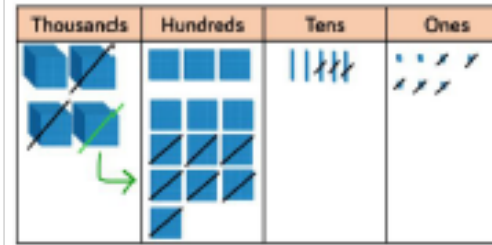
Plain counters on a place value grid can also be used to support learning.



$$435 - 273 = 262$$



$$4,357 - 2,735 = 1,622$$



YEAR 4

Skill: Subtract numbers with up to 4 digits

Base 10 and place value counters are the most effective manipulatives when subtracting numbers with up to 4 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written method.

Plain counters on a place value grid can also be used to support learning.

YEAR 5/6

Skill: Subtract numbers with more than 4 digits

Place value counters or plain counters on a place value grid are the most effective concrete resource when subtracting numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using column method to subtract larger numbers efficiently.

294,382

182,501

?

294,382

182,501

?

$294,382 - 182,501 = 111,881$

HTh	TTh	Th	H	T	O
2	9	3	2	9	2
	8	2	1	8	2
			1	1	8

Number Shapes (Multiplication and Division)

Number shapes support children's understanding of multiplication as repeated addition.

Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication e.g. odd x odd = even, odd x even = odd, even x even = even.

$5 \times 4 = 20$

$4 \times 5 = 20$

$5 \times 4 = 20$

$4 \times 5 = 20$

$18 \div 3 = 6$

YEAR 5

Skill: Subtract with up to 3 decimal places

Place value counters and plain counters on a place value grid are the most effective manipulative when subtracting decimals with 1, 2 and 3 decimal places.

Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this into context when subtracting money and other measures.

2.7 ?

5.43

5.43

2.7

?

$5.43 - 2.7 = 2.73$

Ones	Tenths	Hundredths
1 1 1 1	0.1 0.1 0.1 0.1	0.01 0.01 0.01
1		

Ones	Tenths	Hundredths
1 1 1 1	1 1 1 1 1 1 1 1 1 1	1 1 1
1 1	1 1 1 1 1 1 1 1 1 1	
	1 1 1 1 1 1 1 1 1 1	

0 2 4 6 8 10 12 14 16 18 20 22 24

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Times Tables

YEAR 2

Skill: 2 times table

Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the two times table, using concrete manipulatives to support. Notice how all the numbers are even and there is a pattern in the ones.

Use different models to develop fluency.

YEAR 2

Skill: 5 times table

Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the five times table, using concrete manipulatives to support. Notice the pattern in the ones as well as highlighting the odd, even, odd, even pattern.

Year 2 resources include: a ten frame with 10 white dots on a green background; a number line from 0 to 60 in increments of 5; a string of 10 beads (5 red, 5 blue); four hands; five 5p coins; a ten frame with 10 red dots; a ten frame with 10 yellow dots; a ten frame with 10 red dots; a ten frame with 10 yellow dots; a number line from 0 to 20 with blue arcs showing jumps of 5; and a hundred square with multiples of 5 highlighted in yellow.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

YEAR 2

Skill: 10 times table

Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the ten times table, using concrete manipulatives to support. Notice the pattern in the digits - the ones are always 0, and the tens increase by 1 each time.

Year 3 resources include: a ten frame with 10 white dots on a purple background; a number line from 0 to 100 in increments of 10; a string of 10 beads (5 red, 5 blue); four 10p coins; three ten frames with 10 red dots each; five blue vertical bars; and a hundred square with multiples of 10 highlighted in yellow.

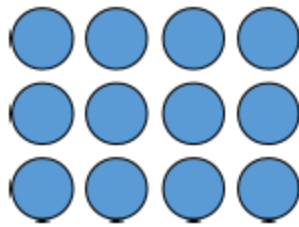
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

YEAR 3

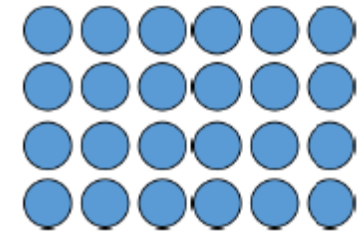
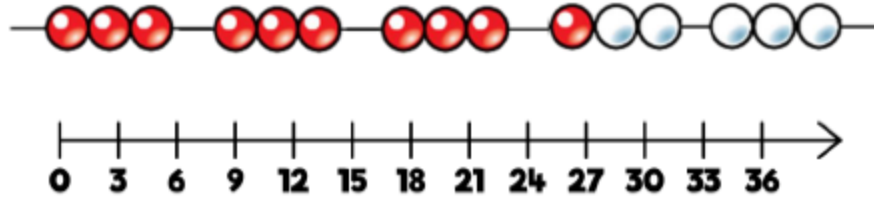
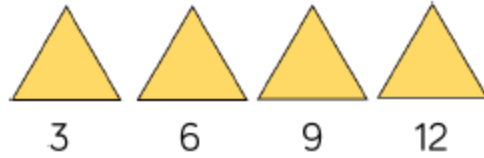
Skill: 3 times tables

Encourage daily counting in multiples both forwards and backward. This can be supported using a number line or a hundred square.

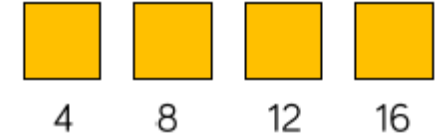
Look for patterns in the three times table, using concrete manipulatives to support. Notice the odd, even, odd, even pattern using number shapes to support. Highlight the pattern in the ones using a hundred square.



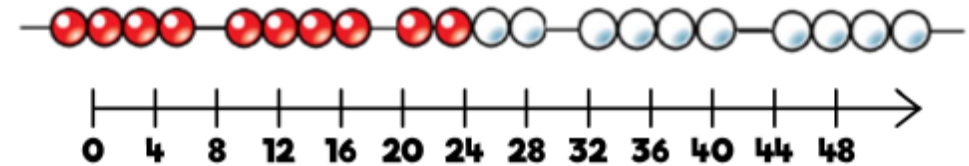
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50



4	8	12	16	20
24	28	32	36	40
44	48	52	56	60



YEAR 3

Skill: 4 times table

Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the four times table, using manipulatives to support. Make links to the 2 times table, seeing how each multiple is double the twos. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

YEAR 4

Skill: 8 times table

Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the eight times table, using manipulatives to support. Make links to the 4 times table, seeing how each multiple is double the fours. Notice the pattern in the ones within each group of five multiple. Highlight that all the multiples are even using number shapes to support.

8 16 24 32

8	16	24	32	40
48	56	64	72	80

0 8 16 24 32 40 48 56 64 72 80 88 96

YEAR 4

Skill: 6 times table

Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the six times table, using manipulatives to support. Make links to the 3 times table, seeing how each multiple is double the threes. Notice the pattern in the ones within each group of five multiples.

6	12	18	24	30
36	42	48	54	60
66	72	78	84	90

0 6 12 18 24 30 36 42 48 54 60 66 72

YEAR 4

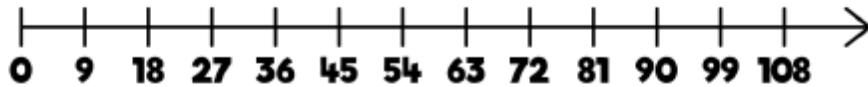
Skill: 9 times table

Encourage daily counting multiples both forwards and backwards. This can be supported using a number line or a hundred square. Look for patterns in the nine times table, using concrete manipulative to support. Notice the pattern in the tens and ones using the hundred square to support as well as noting the odd, even pattern within the multiples.



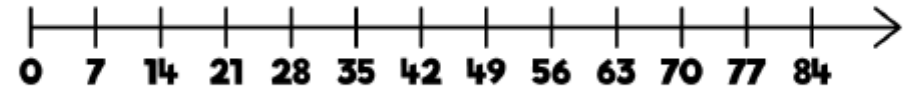
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

9	18	27	36	45
54	63	72	81	90



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

7	14	21	28	35
42	49	56	63	70



YEAR 4

Skill: 7 times table

Encourage daily counting in multiples both forwards and backwards, supported by a number line or a hundred square. The seven times table can be trickier to learn due to the lack of obvious pattern in the numbers, however they already know several facts due to commutativity. Children can still see the odd, even pattern in the multiples using number shapes to support.

YEAR 4

Skill: 11 times table

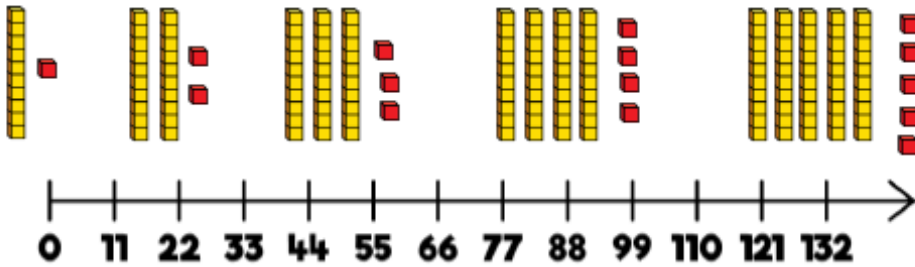
Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the eleven times table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support. Also consider the pattern after crossing 100.

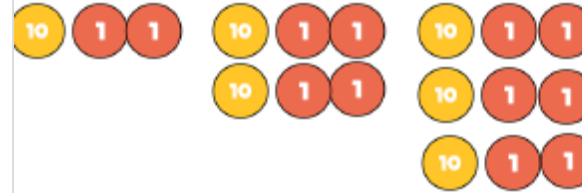
11	22	33	44	55	66
77	88	99	110	121	132



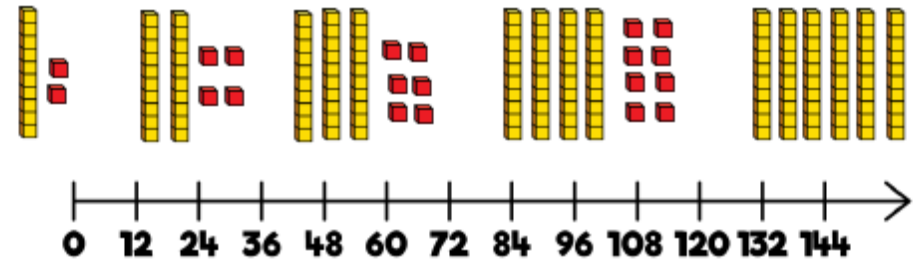
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



12	24	36	48	60
72	84	96	108	120
132	144			



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



YEAR 4

Skill: 12 times table

Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the 12 times table, using manipulatives to support. Make links to the 6 times table, seeing how each multiple is double the sixes. Notice the pattern in the ones within each group of five multiples. The hundred square can support in highlighting this pattern.

Multiplication

YEAR 1/2

Skill: Solve 1-step problems using multiplication

Children represent multiplication as repeated addition in many different ways.

In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record multiplication formally.

In Year 2, children are introduced to the multiplication symbol.

One bag holds 5 apples.
How many apples do 4 bags hold?

$5 + 5 + 5 + 5 = 20$
 $4 \times 5 = 20$
 $5 \times 4 = 20$

	H	T	O
		3	4
×			5
		2	0
+	1	5	0
	1	7	0

$34 \times 5 = 170$

	H	T	O
		3	4
×			5
	1	7	0

YEAR 3/4

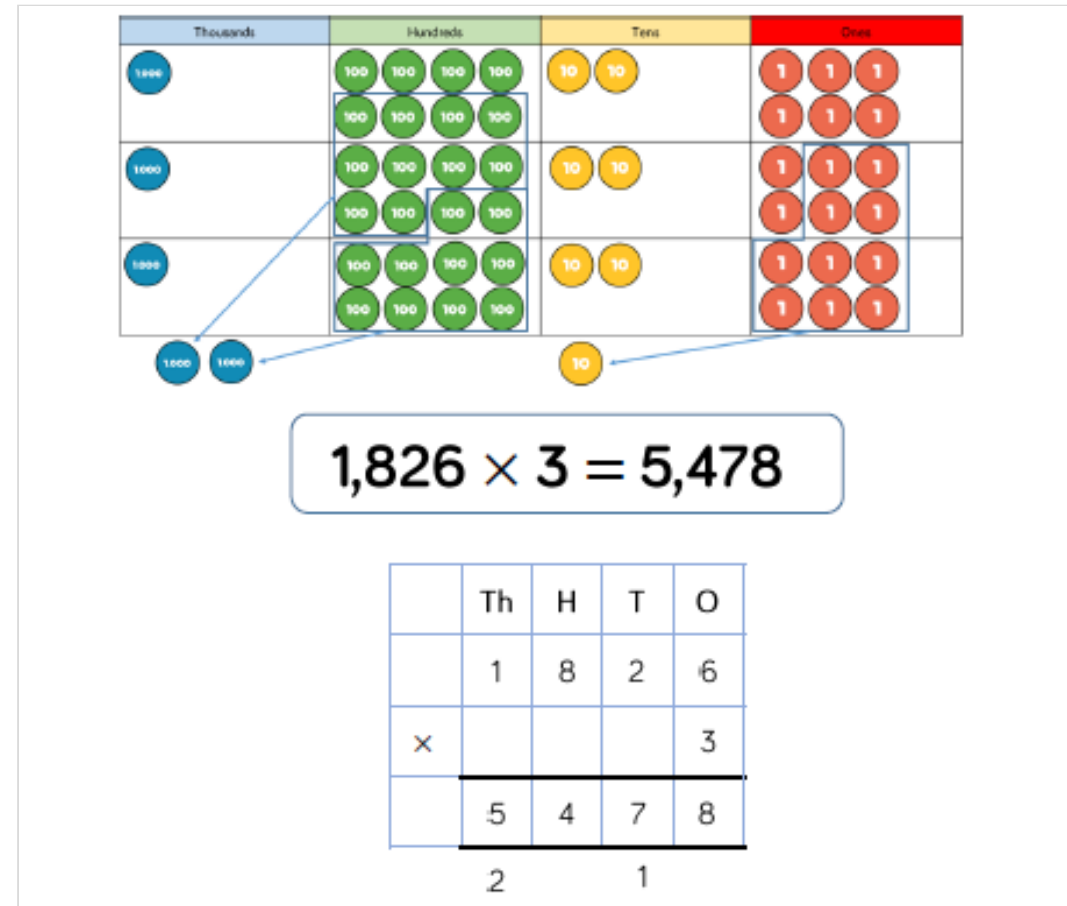
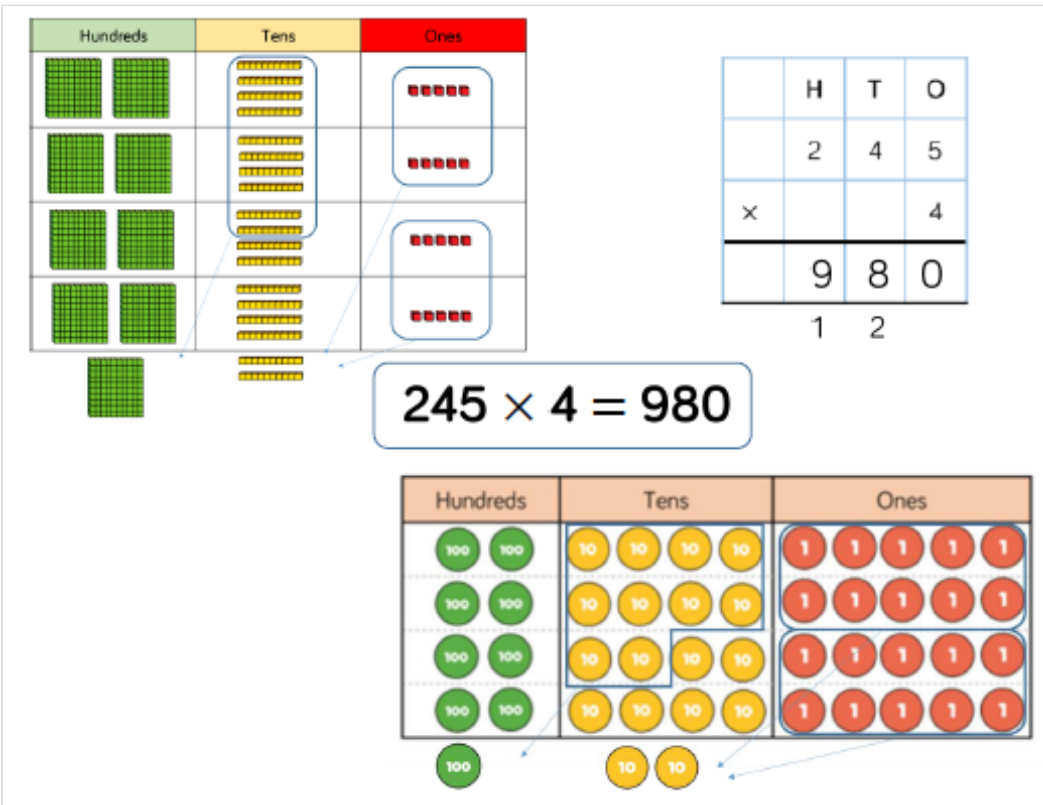
Skill: Multiply 2-digit numbers by 1-digit numbers

Teachers may decide to first look at the expanded column method before moving on to the short multiplication method. The place value counters should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge.

YEAR 3/4

Skill: Multiply 3-digit numbers by 1-digit numbers

When moving to 3-digit by 1-digit multiplication, encourage children to move towards the short, formal written method. Base 10 and place value counters continue to support the understanding of the written method. Limit the number of exchanges needed in the questions and move children away from resources when multiplying larger numbers.



YEAR 5

Skill: Multiply 4-digit numbers by 1-digit numbers

When multiplying 4-digit numbers, place value counters are the best manipulative to use to support children in their understanding of the formal written method. If children are multiplying larger numbers and struggling with their times tables, encourage the use of multiplication grids so children can focus on the use of the written method.

YEAR 5

Skill: Multiply 2-digit numbers by 2-digit numbers

When multiplying a multi-digit number by 2-digits, use the area model to help understand the size of the numbers they are using. This links to finding the area of a rectangle by finding the space covered by the Base 10. The grid method matches the area model as an initial written method before moving on to the formal written multiplication method.

	10	10	1	1
10	100	100	10	10
10	100	100	10	10
10	100	100	10	10
1	10	10	1	1

	H	T	O
		2	2
x		3	1
		2	2
	6	6	0
	6	8	2

x	20	2
30	600	60
1	20	2

$22 \times 31 = 682$

YEAR 5

Skill: Multiply 3-digit numbers by 2-digit numbers

Children can continue to use the area model when multiplying 3-digits by 2-digits. Place value counters become more efficient to use but Base 10 can be used to highlight the size of numbers.

Encourage children to move towards the formal written method, seeing the links with the grid method.

	100	100	10	10	10	1	1	1	1
10	1000	1000	100	100	100	10	10	10	10
10	1000	1000	100	100	100	10	10	10	10
10	1000	1000	100	100	100	10	10	10	10
1	100	100	10	10	10	1	1	1	1
1	100	100	10	10	10	1	1	1	1

	Th	H	T	O	
		2	3	4	
x			3	2	
		4	6	8	
1	7	1	0	2	0
7	4	8	8		

x	200	30	4
30	6,000	900	120
2	400	60	8

$234 \times 32 = 7,488$

YEAR 5/6

Skill: Multiply 4-digit numbers by 2-digit numbers

When multiplying 4-digits by 2-digits, children should be confident in the written method.

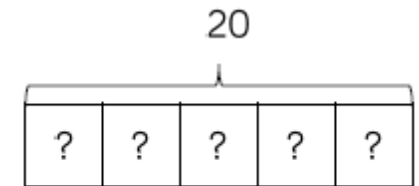
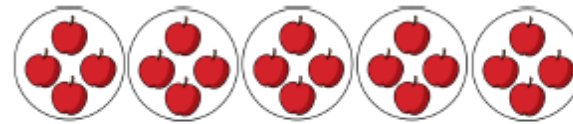
If they are still struggling with times tables, provide multiplication grids to support when they are focusing on the use of the method.

Consider where exchanged digits are placed and make sure this is consistent.

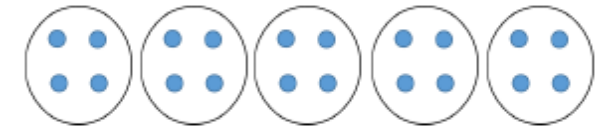
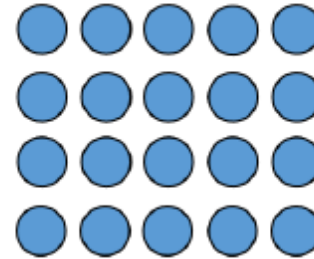
TTh	Th	H	T	O
	2	7	3	9
×			2	8
2	1	9	1	2
₂	₅	₃	₇	
5	4	7	8	0
₁		₁		
7	6	6	9	2

1

$$2,739 \times 28 = 76,692$$



There are 20 apples altogether.
They are shared equally between 5 bags.
How many apples are in each bag?



$$20 \div 5 = 4$$

Division

YEAR 1/2

Skill: Solve 1-step problems using multiplication (sharing)

Children solve problems by sharing amounts into equal groups.

In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record division formally.

In Year 2, children are introduced to the division symbol.

YEAR 1/2

Skill: Solve 1-step problems using division (grouping)

Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They can use concrete representations in fixed groups such as number shapes which helps to show the link between multiplication and division.

There are 20 apples altogether.
They are put in bags of 5.
How many bags are there?

$$20 \div 5 = 4$$

YEAR 1/2

Skill: Divide 2-digits by 1-digit (sharing with no exchange)

When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones.

Straws, Base 10 and place value counters can all be used to share numbers into equal groups.

Part-whole models can provide children with a clear written method that matches the concrete representation.

Tens	Ones
10 10	1 1 1 1
10 10	1 1 1 1

$$48 \div 2 = 24$$

48

40 8

↓ ÷ 2 ↓ ÷ 2

□ □

YEAR 3/4

Skill: Divide 2-digits by 1-digit (sharing with exchange)

When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones. Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows.

Flexible partitioning in a part-whole model supports this method.

52

Tens	Ones
50	2
40	12

$52 \div 4 = 13$

Tens	Ones
40	12
40	12

$40 \div 4 = 10$
 $12 \div 4 = 3$
 $10 + 3 = 13$

Tens	Ones
50	2
40	12

$52 \div 4 = 13$

Tens	Ones
40	12
40	12

YEAR 4/5

Skill: Divide 2-digits by 1-digit (grouping)

When using the short division method, children use grouping. Starting with the largest place value, they group by the divisor.

Language is important here. Children should consider 'How many groups of 4 tens can we make?' and 'How many groups of 4 ones can we make?'

Remainders can also be seen as they are left ungrouped.

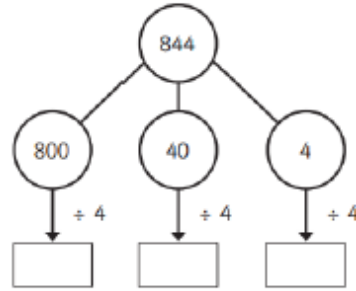
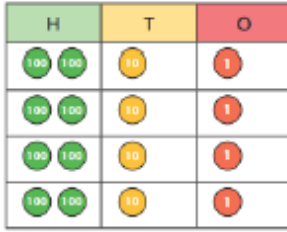
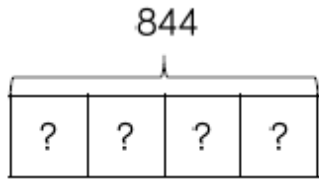
YEAR 4

Skill: Divide 3-digits by 1-digit (sharing)

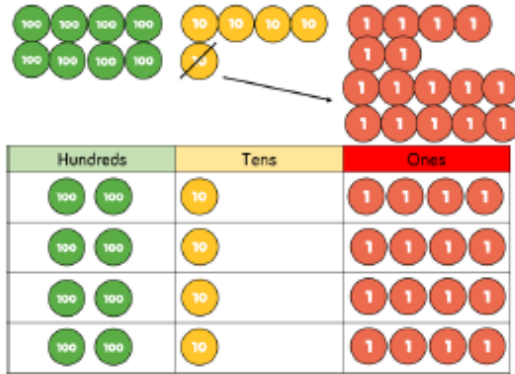
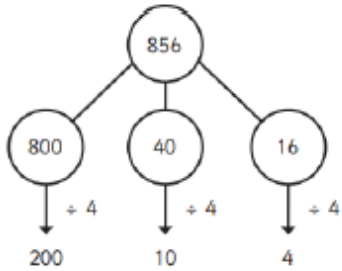
Children can continue to use place value counters to share 3-digit numbers into equal groups. Children should start with the equipment outside the place value grid before sharing the hundreds, tens and ones equally between the rows. This method can also help to highlight remainders.

Flexible partitioning in a part-whole model supports this method.

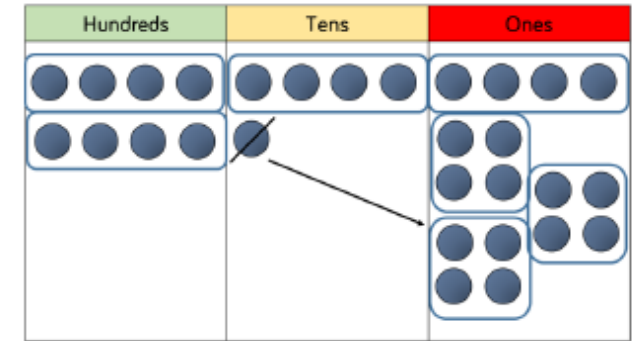
$$844 \div 4 = 122$$



$$844 \div 4 = 122$$



		2	1	4
	4	8	5	16



$$856 \div 4 = 214$$

YEAR 5

Skill: Divide 3-digits by 1-digit (grouping)

Children can continue to use grouping to support their understanding of short division when dividing a 3-digit number by a 1-digit number.

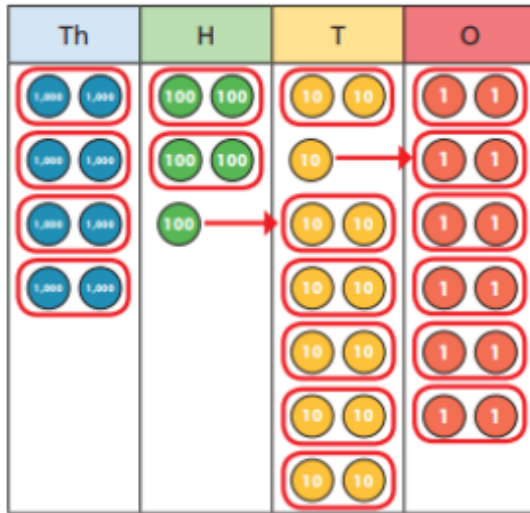
Place value counters or plain counters can be used on a place value grid to support this understanding. Children can also draw their own counters and group them through a more pictorial method.

YEAR 5

Skill: Divide 4-digits by 1-digit (grouping)

Place value counters or plain counters can be used on a place value grid to support children to divide 4-digits by 1-digit. Children can also draw their own counters and group them through a more pictorial method.

Children should be encouraged to move away from the pictorial when dividing numbers with multiple exchanges.



	4	2	6	6
2	8	5	13	12

$$8,532 \div 2 = 4,266$$

YEAR 6

Skill: Divide multi-digits by 2-digits (short division)

When children begin to divide up to 4-digits by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective. Children can write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.

		0	3	6
	12	4	43	72

$$432 \div 12 = 36$$

$$7,335 \div 15 = 489$$

		0	4	8	9
15	7	73	133	135	

15	30	45	60	75	90	105	120	135	150
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YEAR 6

Skill: Divide multi-digits by 2-digits (long division)

Children can also divide by 2-digit numbers using long division.

Children can write out multiples to support their calculations with larger remainders.

Children will also solve problems with remainders where the quotient can be rounded as appropriate.

		0	3	6
1	2	4	3	2
	-	3	6	0
			7	2
	-		7	2
				0

- (x30) $12 \times 1 = 12$
- $12 \times 2 = 24$
- $12 \times 3 = 36$
- $12 \times 4 = 48$
- $12 \times 5 = 60$
- (x6) $12 \times 6 = 72$
- $12 \times 7 = 84$
- $12 \times 8 = 96$
- $12 \times 7 = 108$
- $12 \times 10 = 120$

$$432 \div 12 = 36$$

$$7,335 \div 15 = 489$$

	0	4	8	9
15	7	3	3	5
-	6	0	0	0
	1	3	3	5
-	1	2	0	0
		1	3	5
-		1	3	5
				0

- (x400) $1 \times 15 = 15$
- $2 \times 15 = 30$
- $3 \times 15 = 45$
- (x80) $4 \times 15 = 60$
- $5 \times 15 = 75$
- (x9) $10 \times 15 = 150$

YEAR 6

Skill: Divide multi-digits by 2-digits (long division)

When a remainder is left at the end of a calculation, children can either leave it as a remainder

or convert it to a fraction. This will depend on the context of the question.

Children can also answer questions where the quotient needs to be rounded according to the context.

$$372 \div 15 = 24 \text{ r}12$$

		2	4	r	1	2
1	5	3	7	2		
	-	3	0	0		
		7	2			
	-	6	0			
		1	2			

- $1 \times 15 = 15$
- $2 \times 15 = 30$
- $3 \times 15 = 45$
- $4 \times 15 = 60$
- $5 \times 15 = 75$
- $10 \times 15 = 150$

		2	4	$\frac{4}{5}$
1	5	3	7	2
	-	3	0	0
		7	2	
	-	6	0	
		1	2	

$$372 \div 15 = 24 \frac{4}{5}$$
